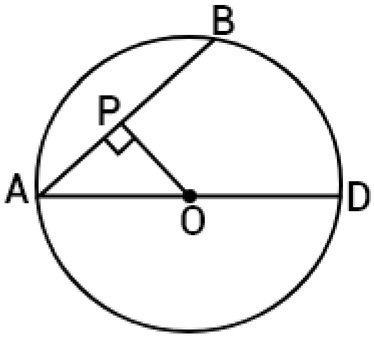
Chapter 10
Circles

Exercise No. 10.1

Multiple Choice Questions:

1. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is:
(A) 17 cm
(B) 15 cm
(C) 4 cm
(D) 8 cm

Solution:
Construction: Draw OP $\perp$ AB.



As perpendicular from the center to a chord bisect. So,

$$AP = \frac{1}{2} \times AB = \frac{1}{2} \times 30 = 15\text{cm}$$

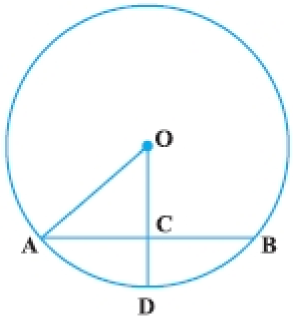
$$\text{Radius} = OA = \frac{1}{2} \times 34 = 17\text{cm}$$

Now, in right triangle OPA,

$$OP = \sqrt{OA^2 - AP^2}$$
$$= \sqrt{(17)^2 - (15)^2}$$
$$= \sqrt{289 - 225}$$
$$= \sqrt{64}$$
$$= 8\text{cm}$$

Hence, the correct option is (D).

2. In Fig., if OA = 5 cm, AB = 8 cm and OD is perpendicular to AB, then CD is equal to:



(A) 2 cm
(B) 3 cm
(C) 4 cm
(D) 5 cm

Solution:
As the perpendicular from the centre of a circle to a chord bisects the chord.

AC = CB = $\frac{1}{2}$ x AB = $\frac{1}{2}$ x 8 = 4 cm

Given OA = 5 cm

$AO^2 = AC^2 + OC^2$

$(5)^2 = (4)^2 + OC^2$

$25 = 16 + OC^2$

$OC^2 = 25 - 16$

$= 9$

So, OC = 3 cm [Length is always positive]
OA = OD [same radius of a circle]
OD = 5 cm
CD = OD – OC
= 5 – 3
= 2 cm
Hence, the correct option is (A).

3. If AB = 12 cm, BC = 16 cm and AB is perpendicular to BC, then the radius of the circle passing through the points A, B and C is :
(A) 6 cm
(B) 8 cm
(C) 10 cm
(D) 12 cm

Solution:
Given in the question, AB = 12 cm and BC = 16 cm.
In a circle, $BC \perp AB$. So, that means AC will be a diameter of circle.

Now, by using Pythagoras theorem in right angled triangle ABC.

$AC^2 = AB^2 + BC^2$

$AC^2 = (12)^2 + (16)^2$

$AC^2 = 144 + 256$

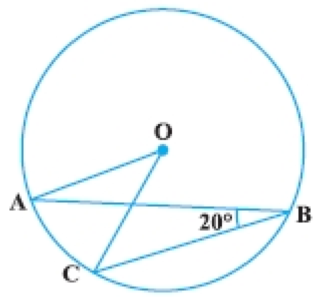
$AC^2 = 400$

$AC = 20\text{cm}$

So, radius of circle = $\frac{1}{2} \times AC = \frac{1}{2} \times 20 = 10\text{cm}$.

Therefore, the radius of circle is 10cm.
Hence, the correct option is (C).

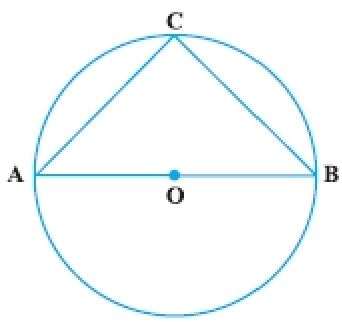
4. In Fig., if ∠ABC = 20º, then ∠AOC is equal to:



(A) 20º
(B) 40º
(C) 60º
(D) 10º

Solution:
Given: $\angle ABC = 20°$
As angle subtended at the centre by an arc is twice the angle subtended by it at the remaining part of circle. So,
$\angle AOC = 2\angle ABC = 2 \times 20° = 40°$.
Hence, the correct option is (B).

5. In Fig., if AOB is a diameter of the circle and AC = BC, then ∠CAB is equal to:



(A) 30º

(B) 60º
(C) 90º
(D) 45º

Solution:
Given: AOB is a diameter of the circle and AC = BC.
So, $\angle C = 90°$ [Angle on the semicircle is $90°$]
Now, AC = BC
So, $\angle A = \angle B$ [Angles opposite to equal sides of triangle are equal]

Now, by using the sum property of a triangle,
$\angle A + \angle B + \angle C = 180°$

$2\angle A + 90° = 180°$

$2\angle A = 180° - 90°$

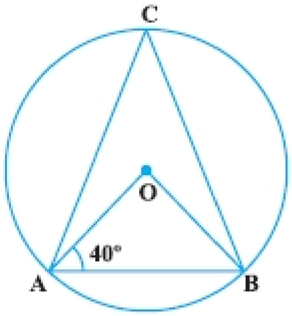
$2\angle A = 90°$

$\angle A = \frac{90°}{2}$

$\angle A = 45°$

Hence, the correct option is (D).

6. In Fig., if ∠OAB = 40º, then ∠ACB is equal to:



(A) 50º
(B) 40º
(C) 60º
(D) 70°

Solution:
Given: ∠OAB = 40º

Now, in triangle OAB,
OA = OB [Radii of circle]
So, $\angle OAB = \angle OBA = 40°$ [Angle opposite to equal sides are equal]
So,
$\angle AOB = 180° - (40° + 40°)$

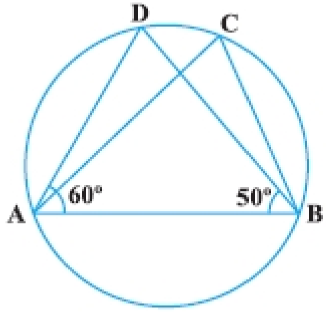
$= 100°$

As we know that angle subtended by an arc of a circle at the center is double the angle subtended by it at any point on the remaining part of the circle. So,

$\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2}\times 100° = 50°$

Hence, the correct option is (A).

7. In Fig., if ∠DAB = 60º, ∠ABD = 50º, then ∠ACB is equal to:



(A) 60º
(B) 50º
(C) 70º
(D) 80º

Solution:

In triangle ABC,

$\angle A + \angle B + \angle D = 180°$ [Angle sum property of a triangle]

$60° + 50° + \angle D = 180°$

$\angle D = 180° - 110°$

$\angle D = 70°$

That is $\angle ABD = 70°$

Now, $\angle ACB = \angle ADB = 70°$ [Angle in the same segment of a circle are equal]

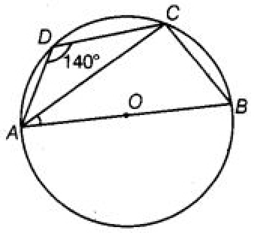
Hence, the correct option is (C).

8. ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and
∠ADC = 140º, then ∠BAC is equal to:
(A) 80º
(B) 50º
(C) 40º
(D) 30º

Solution:

Given: ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and ∠ADC = 140º.

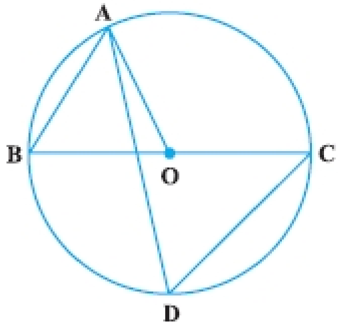
Construction: Join AC.



See in the figure,
$\angle ADC + \angle ABC = 180°$ [Given]
$140° + \angle ABC = 180°$
So, $\angle ABC = 180° - 140° = 40°$
Now, $\angle C = 90°$ [Angle in semicircle is a right angle]

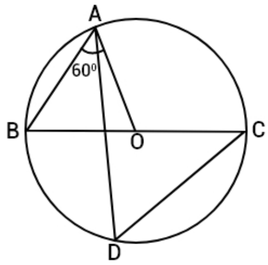
In triangle ABC,
$\angle BAC = 180° - (90° + 40°)$
$= 50°$
Hence, the correct option is (B).

9. In Fig., BC is a diameter of the circle and ∠BAO = 60º. Then ∠ADC is equal to:



(A) 30º
(B) 45º
(C) 60º
(D) 120º

Solution:
Given: BC is a diameter of the circle and ∠BAO = 60º.

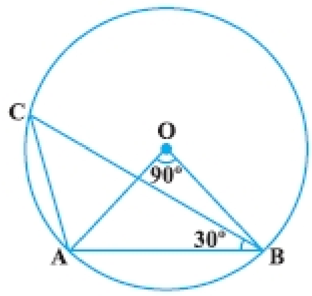


Now, in triangle OAB,
OA = OB [Radii of the same circle]
So, $\angle ABO = \angle BAO$ [Angle opposite to equal sides are equal]

$\angle ABO = \angle BAO = 60°$ [Given]

Now, $\angle ADC = \angle ABC = 60°$ [$\angle ADC$ and $\angle ABC$ are angles in the same segment of a circle are equal]
Therefore, $\angle ADC = 60°$.
Hence, the correct option is (C).

10. In Fig. 10.9, ∠AOB = 90º and ∠ABC = 30º, then ∠CAO is equal to:



(A) 30º
(B) 45º
(C) 90º
(D) 60º

Solution:
In triangle OAB,
OA = OB [Radii of the same circle]
So, $\angle OAB = \angle OBA$

Now, in triangle OAB,
$\angle OAB + \angle OBA + \angle AOB = 180°$
So,
$2\angle OAB = 180° - \angle AOB$
$= (180° - 90°)$
$= 90°$ [Sum of angle of triangle is $180°$]
So, $\angle OAB = \frac{1}{2} \times 90° = 45°$
Also, $\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 90° = 45°$

Now, in triangle CAB,
$\angle CAB = 180° - (\angle ABC + \angle ACB)$
$= 180° - (30° + 45°) = 105°$

Now, $\angle CAO = \angle CAB - \angle OAB$
$\angle CAO = 105° - 45°$
Hence, the correct option is (D).

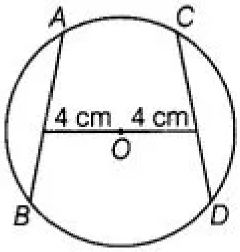
Exercise No. 10.2

Short Answer Questions with Reasoning:

Write True or False and justify your answer in each of the following:

1. Two chords AB and CD of a circle are each at distances 4 cm from the center. Then AB = CD.

Solution:
As we know that the chords equidistant from the centre of circle are equal in length.



Hence, the given statement is true.

2. Two chords AB and AC of a circle with center O are on the opposite side of OA. Then $\angle OAB = \angle OAC$.

Solution:
In this question, two chords AB and AC are not given equal.
Hence, the given statement is false because the angles will be equal if AB = AC.

3. Two congruent circles with center's O and O′ intersect at two points A and B. Then $\angle AOB = \angle AO'B$.

Solution:
The equal chords of congruent circle subtend equal angles at the respective centers.
Hence, the given statement is true.

4. Through three collinear points a circle can be drawn.

Solution:
A circle can pass through only two collinear points but not through three collinear points.
Hence, the given statement is false.

5. A circle of radius 3 cm can be drawn through two points A, B such that AB = 6 cm.

Solution:
As we know that radii of circle is half of the diameter. So,

Radii of circle $= \frac{6}{2}\text{cm} = 3\text{cm}$

Hence, the given statement is true.

6. If AOB is a diameter of a circle and C is a point on the circle, then $AC^2 + BC^2 = AB^2$.

Solution:
Given: AOB is a diameter of a circle and C is a point on the circle.
So, $\angle ACB = 90°$ [Angle in a semicircle is a right angle]

In right triangle ABC,
$AC^2 + BC^2 = AB^2$ [By Pythagoras theorem]
Hence, the correct option is true.

7. ABCD is a cyclic quadrilateral such that $\angle A = 90°$, $\angle B = 70°$, $\angle C = 95°$ and $\angle D = 105°$.

Solution:
Given: ABCD is a cyclic quadrilateral such that $\angle A = 90°$, $\angle B = 70°$, $\angle C = 95°$ and $\angle D = 105°$.

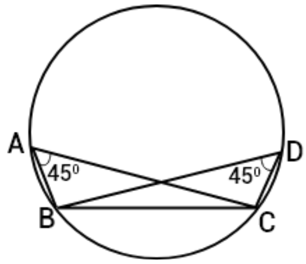
Now, sum of the opposite site of angle of quadrilateral is:
$\angle A + \angle C = 90° + 95° = 185°$
And, $\angle B + \angle D = 70° + 105° = 175°$
Since, sum of opposite angles is not equal to $180°$. So, ABCD is not a cyclic quadrilateral.
Hence, the given statement is true.

8. If A, B, C, D are four points such that $\angle BAC = 30°$ and $\angle BDC = 60°$, then D is the center of the circle through A, B and C.

Solution:
There can be many points D, such that ∠BDC = 60° and each such point cannot be the centre of the circle through A, B and C.
Hence, the given statement is false.

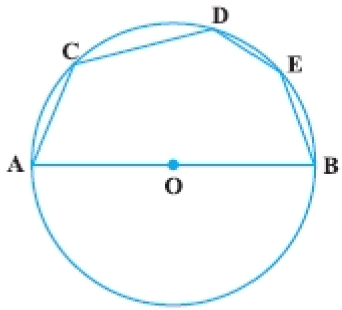
9. If A, B, C and D are four points such that $\angle BAC = 45°$ and $\angle BDC = 45°$, then A, B, C, D are concyclic.

Solution:
Given: $\angle BAC = 45°$ and $\angle BDC = 45°$

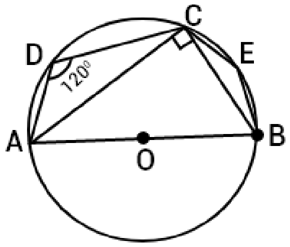


As we know that, angles in the same segment of a circle are equal. Hence, A, B, C and D are concyclic.
Hence, the given statement is true.

10. In Fig., if AOB is a diameter and ∠ADC = 120°, then ∠CAB = 30°.



Solution:
See the given figure, AOB is a diameter of circle with center O.
$\angle ADC + \angle ABC = 180°$ [ABCD is a cyclic quadrilateral]
$120° + \angle ABC = 180°$



$\angle ABC = 180° - 120° = 60°$

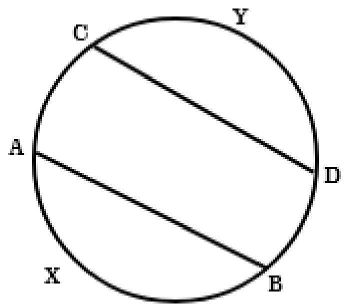
In triangle ABC,
$\angle ACB = 90°$ [Angle in a semicircle and $\angle ABC = 60°$ (proved above)]
So, $\angle CAB = 180° - (90° + 60°) = 30°$
Hence, the given statement is true.

Exercise No. 10.3

Short Answer Questions:

1. If arcs AXB and CYD of a circle are congruent, find the ratio of AB and CD.

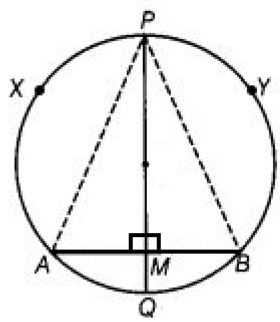
Solution:
As we know that if two arcs of a circle are congruent, then their corresponding arcs are equal.
So, we have chord AB = chord CD.



Hence, the ratio of AB and CD is 1:1.

2. If the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q, prove that $\text{arcPXA} \cong \text{arcPYB}$**.**

Solution:
Given: PQ is the perpendicular bisector of AB.
To prove that $\text{arcPXA} \cong \text{arcPYB}$.



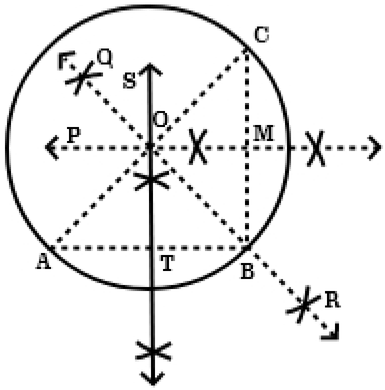
Proof: See the above figure,
AM = BM

In triangle APM and triangle BPM,
AM = BM [Proved above]
$\angle AMP = \angle BMP$ [Each = $90°$]
PM = PM [Common side]
So, $\Delta APM \cong \Delta BPM$ [By SAS congruence rule]

So, AP = BP   [By CPCT]
Hence, $\text{arc}PXA \cong \text{arc}PYB$.   [If two chords of a circle are equal, then their corresponding arcs are congruent]

3. A, B and C are three points on a circle. Prove that the perpendicular bisectors of AB, BC and CA are concurrent.

Solution:
Given: A, B and C are three points on a circle.



To prove that perpendicular bisector of AB, BC and CA.

Construction: Draw perpendicular bisectors ST of AB, PM of BC and QR of CA. Join AB, BC, and CA.

Proof: OA = OB   …(I) [O lies on ST, the perpendicular bisector of AB]
Again, OB = OC …(II) [O lies on PM the perpendicular bisector of BC]
And, OC = OA …(III) [O lies on QR, the perpendicular bisector of CA]

Now, from equation (I), (II), and (III),
OA = OB = OC = r
So, draw a circle with center O and radius r, that will pass through A, B and C.

That means a circle passing through the point A, B and C. Since, ST, PM or QR can cut each other at one and only one point O.
Therefore, O is the only one point which is equidistance from A, B and C.
Hence, the perpendicular bisector of AB, BC and CA are concurrent.

4. AB and AC are two equal chords of a circle. Prove that the bisector of the angle BAC passes through the center of the circle.

Solution:
Given: AB and AC are two equal chords of a circle.
To prove that AM passing through O.
Construction: Let AM intersect BC at P. Join BC.

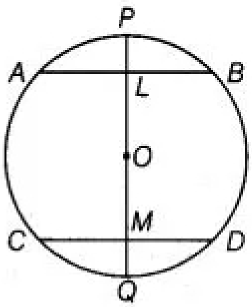
Proof: In triangle BAP and triangle CAP.
AB = AC [Given]
$\angle BAP = \angle CAP$ [Given]

And, AP = BP [Common side]
So, $\Delta BAP \cong \Delta CAP$ [By SAS congruency]
Again, $\angle BAP = \angle CPA$ [CPCT]
And, CP = PB
But, $\angle BPA + \angle CPA = 180°$ [linear pair angles]
Now, $\angle BPA = \angle CPA = 90°$

Since, AP is perpendicular bisector of the chord BC, which will pass through the center O on being produced.
Hence, AM passes through O.

5. If a line segment joining mid-points of two chords of a circle passes through the center of the circle, prove that the two chords are parallel.

Solution:
Given: The diameter PQ passes through the center O of the circle. AB and CD are two chords of a circle whose center of O, and PQ is a diameter bisecting the chord AB and CD at L and M respectively.



To prove that AB||CD
Proof: See the figure, the mid-point of AB is L.
So, OL ⊥ AB [The line joining the center of circle to the mid-point of a chord is perpendicular to the chord]
$\angle ALO = 90°$ … (I)

Again, OM ⊥ CD
So, $\angle OMD = 90°$

Now, from equation (I) and (II), get:
$\angle ALO = \angle OMD = 90°$ (II)
Since, there are alternating angles. So,
AB||CD
Hence proved.

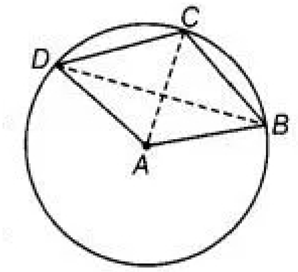
6. ABCD is such a quadrilateral that A is the centre of the circle passing through B, C and D. Prove that

$\angle CBD + \angle CDB = \frac{1}{2}\angle BAD$

Solution:

In a circle, ABCD is a quadrilateral having center A.

To prove $\angle CBD + \angle CDB = \frac{1}{2}\angle BAD$



Construction: join AC.

Proof: As we know that angle subtended by an arc at the center is double the angle subtended by it at point on the remaining part of the circle.

So, $\angle CAD = 2\angle CBD$ ...(I)

And $\angle BAC = 2\angle CDB$ ...(II)

Now, adding equation (I) and (II), get:

$\angle CAD + \angle BAC = 2(\angle CBD + \angle CDB)$

$\angle BAD = 2(\angle CBD + \angle CDB)$

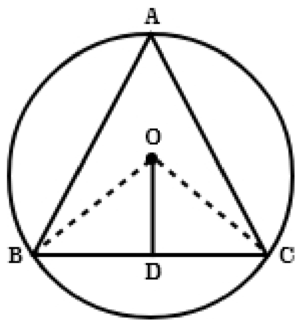
Hence, $\angle CBD + \angle CDB = \frac{1}{2}\angle BAD$.

7. O is the circumventer of the triangle ABC and D is the mid-point of the base BC. Prove that ∠ $\angle BOD = \angle A$

Solution:

Given: $OD \perp BC$ and O is the circumcenter of ΔABC.

To prove that $\angle BOD = \angle A$



Construction: join OD and OC.

Proof: In triangle OBD and triangle OCD,

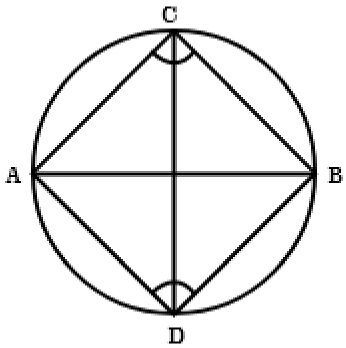
OB = OC [Each equal to radius of the circumcircle]

$\angle ODB = \angle ODC$ [Each of $90°$]
OD = OD [Common]
So, $\angle OBD \cong \angle OCD$ [By SAS congruency]

Since, $\angle BOD = \angle COD$ [By CPCT]
$\angle BOC = 2\angle BOD = 2\angle COD$
Therefore, $\angle BOC = 2\angle A$
Now, $2\angle BOD = 2\angle A$ [$\angle BOC = 2\angle BOD$]
$\angle BOD = \angle A$
Hence, proved.

8. On a common hypotenuse AB, two right triangles ACB and ADB are situated on opposite sides. Prove that $\angle BAC = \angle BDC$.

Solution:
To prove that $\angle BAC = \angle BDC$.
Proof: In right triangle ACB and ADB,

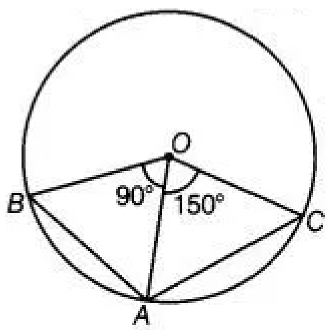


$\angle ACB = 90°$ and $\angle ADB = 90°$
So, $\angle ACB + \angle ADB = 90° + 90° = 180°$

As we know that if the sum of any pair of opposite angle of a quadrilateral is $180°$, then the quadrilateral is cyclic. So, ADBC is a cyclic quadrilateral.
Join CD. Now, angle $\angle BAC$ and $\angle BDC$ are made by arc BC in the same segment BDAC.
Hence, $\angle BAC = \angle BDC$. [Angle in the same segment are equal]

9. Two chords AB and AC of a circle subtends angles equal to 90º and 150º, respectively at the center. Find $\angle BAC$, if AB and AC lie on the opposite sides of the center.

Solution:
In triangle BOA,
OB = OA [Both are the radius of circle]
$\angle OAB = \angle OBA$ … (I) [Angle opposite to equal sides are equal]



Now, In triangle OAB,

$\angle OBA + \angle OAB + \angle AOC = 180°$ [By angle sum property of a triangle]

$\angle OAB + \angle OAB + 90° = 180°$ [From equation (I)]

$2\angle OAB = 180° - 90°$

$2\angle OAB = 90°$

$\angle OAB = 45°$

Again, in triangle AOC,

AO = OC [radii or circle]

$\angle OCA = \angle OAC \quad \ldots(II)$ [Angle opposite to equal sides are equal]

Now, by angle sum property of a triangle:

$\angle AOC + \angle OAC + \angle OCA = 180°$

$150° + 2\angle OAC = 180°$ [From equation (II)]

$2\angle OAC = 180° - 150°$

$2\angle OAC = 30°$

$\angle OAC = 15°$

$\angle BAC = \angle OAB + \angle OAC = 45° + 15° = 60°$

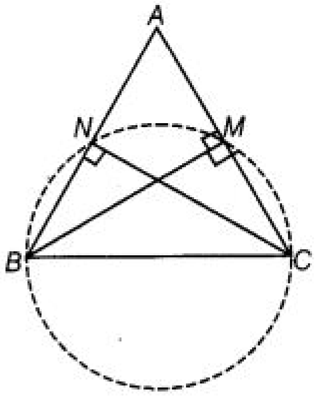
10. If BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC, prove that the points B, C, M and N are concyclic.

Solution:

Given: In ΔABC, BM ⊥ AC and CN ⊥ AB.

To prove that points B, C, M and N are con-cyclic.

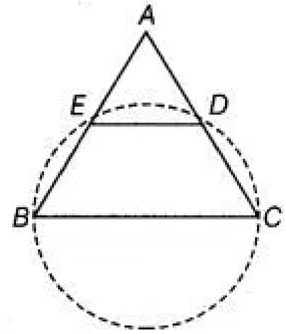
Construction: Draw a circle passing through the points B, C, M and N.



Proof suppose, we consider SC as a diameter of the circle. Also, we know that SC subtends a 90° to the circle.
So, the points M and N should be on a circle.
Hence, BCMN form a con-cyclic quadrilateral.
Hence proved.

11. If a line is drawn parallel to the base of an isosceles triangle to intersect its equal sides, prove that the quadrilateral so formed is cyclic.

Solution:
Given: In triangle ABC is an isosceles triangle such that AB = AC and also DE||SC.
To prove that quadrilateral BCDE is a cyclic quadrilateral.
Construction: Draw a circle passes through the point B, C, D and E.

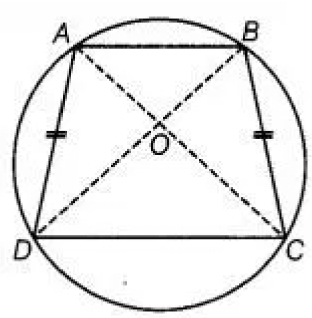


Proof: In triangle ABC:
AB = AC [Equal sides of an isosceles triangle]
$\angle ACB = \angle ABC$ ...(I) [Angles opposite to the equal sides are equal]
Now, DE||BC
$\angle ADE = \angle ACB$ [Corresponding angles] ...(II)

Now, adding equation (I) and (II), get:
$\angle ADE + \angle EDC = \angle ACB + \angle EDC$
$180° = \angle ACB + \angle EDC$ [$\angle ADE$ and ∠EDC from linear pair aniom]
$\angle EDC + \angle ACB = 180°$ [From equation (I)]
Hence, BCDE is a cyclic quadrilateral because sum of the opposite angles is $180°$.

12. If a pair of opposite sides of a cyclic quadrilateral are equal, prove that its diagonals are also equal.

Solution:
Given: let ABCD be a cyclic quadrilateral and AD = BC.



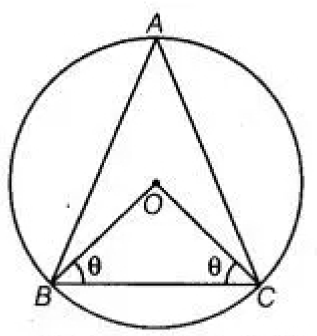
Construction: Join AC and BD.
To prove that AC = BD

Proof: In triangle AOD and triangle BOC,
$\angle OAD = \angle OBC$ and $\angle ODA = \angle OCB$ [Same segments subtends equal angle to the circle]
AB = BC [Given]
$\Delta AOD = \Delta BOC$ [By ASA congruence rule]

Now, adding ΔDOC on both sides, get:
$\Delta AOD + \Delta DOC \cong \Delta BOC + \Delta DOC$
$\Delta ADC \cong \Delta BCD$
AC = BD [By CPCT]
Hence, proved.

13. The circumcentre of the triangle ABC is O. Prove that $\angle OBC + \angle BAC = 90°$

Solution:
Given: A circle is circumscribed on a triangle ABC having center O.



To prove that $\angle OBC + \angle BAC = 90°$
Construction: Join BO and CO.
Proof: Suppose $\angle OBC = \angle OCB = \theta$

Now, in triangle OBC, $\angle BOC + \angle OCB + \angle CBO = 180°$ [By angle sum property of a triangle is $180°$]

$\angle BOC+\theta+\theta=180°$

$\angle BOC=180°-2\theta$

As we know that, in a circle, the angle subtended by an arc at the center is twice the angle subtended by it at the remaining part of the circle.

$$\angle BOC=2\angle BAC$$

$$\angle BAC=\frac{\angle BOC}{2}$$

$$=\frac{180°-2\theta}{2}$$

$$=90°-\theta$$

$$\angle BAC+\theta=90°$$

$\angle BAC+\angle OBC=90°$

Hence, proved.

14. A chord of a circle is equal to its radius. Find the angle subtended by this chord at a point in major segment.

Solution:

Given: AB is a chord of a circle, which is equal to the radius of the circle that is:

AB = BO …(I)

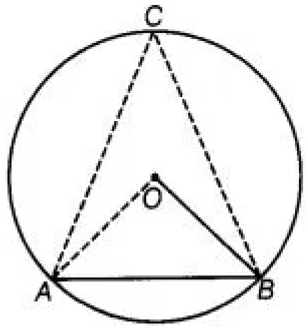
Construction: Join OA, AC and BC

As, OA = OB = Radius of circle

So, OA = AS = BO

Therefore, triangle OAB is an equilateral triangle,

$\angle AOB=60°$ [Each angle of an equilateral triangle is $60°$.



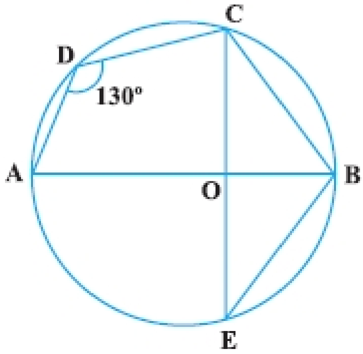
As using the theorem, in a circle, the angle subtended by an arc at the center is twice the angle subtended by it at the remaining part of the circle. So,

$\angle AOB=2\angle ACB$

$\angle ACB=\frac{60°}{2}=30°$

Hence, the angle subtended by this chord at a point in major segment is $30°$.

15. In Fig., ∠ADC = 130° and chord BC = chord BE. Find ∠CBE.



Solution:

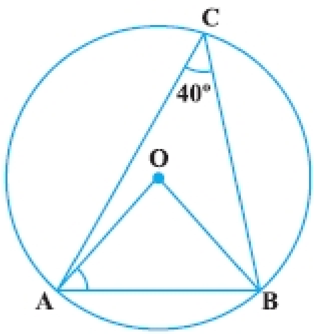
Given: ∠ADC = 130° and chord BC = chord BE.
Let the points A, B, C and D form a cyclic quadrilateral.

As, the sum of opposite angles of a cyclic quadrilateral ΔDCB is 180°.
∠ADC + ∠OBC = 180°
130°+ ∠OBC = 180°
∠OBC = 180° -130° = 50°

In triangle BOC and triangle BOE,
BC = BE [given equal chord]
OC =OE [both are the radius of the circle]

And OB = OB [common side]
ΔBOC ≅ΔBOE
∠OBC = ∠OBE = 50° [by CPCT]
∠CBE = ∠CBO + ∠EBO = 50° + 50° = 100°
Hence, the angle ∠CBE is 100°.

16. In Fig., ∠ACB = 40°. Find ∠OAB.



Solution:

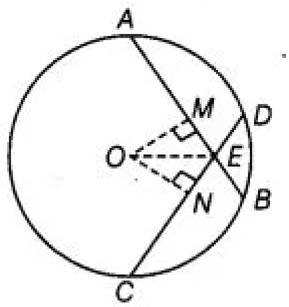
Given: $\angle ACB = 40°$
As we know that, a segment subtends an angle to the circle is half the angle subtends to the centre.
$\angle AOB = 2\angle ACB$

Exercise No. 10.4

Long Answer Questions:

1. If two equal chords of a circle intersect, prove that the parts of one chord are separately equal to the parts of the other chord.

Solution:
Given: let AB and CD are two equal chords of a circle that are meet at point E.
TP prove that:
(1) AE = CE
(2) BE = DE
Construction: Draw OM $\perp$ AB and ON $\perp$ CD and join OE where O is the center of circle.



Proof: In triangle OME and triangle ONE,
OM = ON [Equal chords of equidistance from the centre]
OE = OE [Common side]
$\angle OME = \angle ONE$ [Each $90°$]
So, $\Delta OME \cong \Delta ONE$ [By RHS congruence rule]
EM = EN [By CPCT] …(I)
AB = CD

Now, dividing both side by 2 in the above equation, get:
$\frac{AB}{2} = \frac{CD}{2}$
$AM = CN$ …(II) [Perpendicular drawn from centre to chord bisect the chord that is AM =MB and CN =ND]

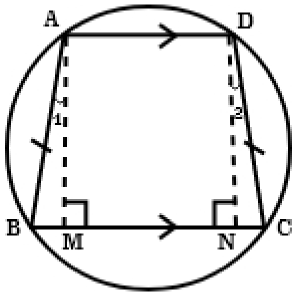
Now, adding equation (I) and (II), get:
EM + AM = EN + CN
AE = CE …(II) [Prove part (1)]
AB = CD

Now, subtracting both sides by AE, get:
AB – AE = CD – AE
BE = CD – CE [From equation (II)]

BE = DE   [Prove part (2)]
Hence, proved.

2. If non-parallel sides of a trapezium are equal, prove that it is cyclic.

Solution:
Given: ABCD is a trapezium which AD||BC and whose non-parallel sides AD and BC are equal that is AB = DC



To prove that trapezium ABCD is cyclic.
Construction: Draw $AM \perp BC$ and $DN \perp BC$.

Proof: In right triangle AMB and DNC.
$\angle AMB = \angle DNC$ [Each $90°$]
AB = DC   [Given]
AM = DN [Perpendicular distance between two parallel lines are same]
$\Delta AMB \cong \Delta DNC$ [By RHS congruence rule]
$\angle B = \angle C$ [By CPCT]
Also, BM = CN   [By CPCT]

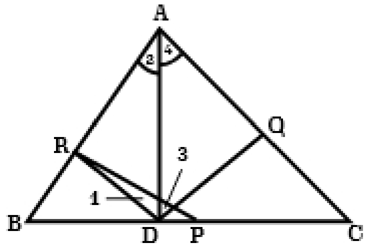
Therefore, $\angle 1 = \angle 2$ [Angles opposite to equal sides are equal]
So, $\angle BAD = \angle 1 + 90°$
$\angle BAD = \angle 2 + 90°$ [$\angle 1 = \angle 2$ Prove above]
$= \angle CDA$

Now, in quadrilateral ABCD,
$\angle B + \angle C + \angle CDA + \angle BAD = 360°$
$\angle B + \angle C + \angle CDA + \angle BAD = 360°$ [As, $\angle B = \angle C$ and $\angle CDA = \angle BAD$ prove above]
$2(\angle B + \angle CDA) = 360°$
$\angle B + \angle CDA = 180°$
As, if the same of any pair of opposite angles of a quadrilateral is $180°$ then the quadrilateral is cyclic.
Hence, the trapezium ABCD is cyclic.

3. If P, Q and R are the mid-points of the sides BC, CA and AB of a triangle and AD is the perpendicular from A on BC, prove that P, Q, R and D are concyclic.

Solution:

To prove that R, D, P and Q are concyclie.



Construction: Join RD, QD, PR and PQ. So, RP join to R and P, the mid-point of AB and BC.

Proof: RP||AC [Mid-point theorem]

Also, PQ||AB

Now, ARPQ is a parallelogram. So,

$\angle RAQ = \angle RPQ$ [opposite angles of a parallelogram] …(I)

In triangle ABD, angle D is right angle and DR is a median. So,

RA = DR and $\angle 1 = \angle 2$ …(II)

Also, $\angle 3 = \angle 4$ …(III)

Now, adding equation (II) and (III), get:

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle RDQ = \angle RAQ$

$= \angle RPQ$

Hence, R, D, P and Q are concyclie.

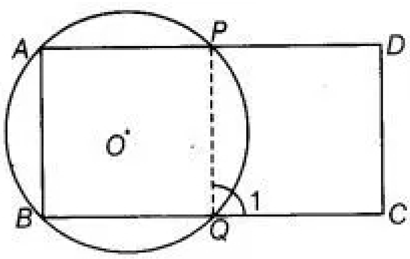
4. ABCD is a parallelogram. A circle through A, B is so drawn that it intersects AD at P and BC at Q. Prove that P, Q, C and D are concyclic.

Solution:

Given: ABCD is a parallelogram. A circle through A, B is so drawn that it intersects AD at P and BC at Q.

A circle through A, B is so drawn that it intersects AD at P and BC at Q.

Construction: Join PQ.



Proof:

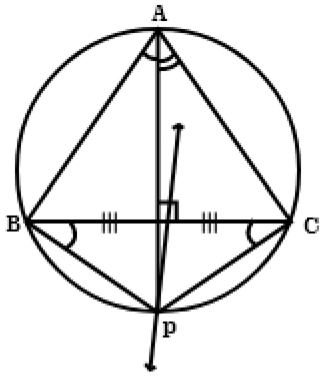
$\angle 1 = \angle A$ [Exterior angle property of cyclic quadrilateral]

Also, $\angle A = \angle C$ [Opposite angles of a parallelogram]
So, $\angle 1 = \angle C$ ... (I)

As, $\angle C + \angle D = 180°$ [sum of cointerior angles on same side is 180°]
$\angle 1 + \angle D = 180°$ **[from equation (I)]**
Since, the quadrilateral QCDP is cyclic.
Therefore, the points P, Q, C and D are con-cyclic.
Hence proved.

5. Prove that angle bisector of any angle of a triangle and perpendicular bisector of the opposite side if intersect, they will intersect on the circumcircle of the triangle.

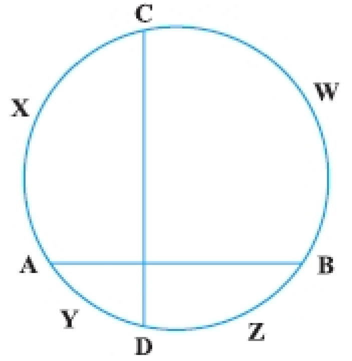
Solution:
Given: In triangle ABC, l is perpendicular bisector of BC.
To prove that angle bisector of $\angle A$ and perpendicular bisector of BC intersect on the circumcircle of triangle ABC.



Proof: See the figure, the angle bisector of $\angle A$ intersect circumcircle of triangle ABC at D.
Join BP and CP.
$\angle BAP = \angle BCP$ [Angle in the same segment are equal]
$\angle BAP = \angle BCP = \frac{1}{2}\angle A$ ...(I) [AP is bisector of $\angle A$]
Also,
$\angle PAC = \angle PBC = \frac{1}{2}\angle A$ ...(II)

Now, from equation (I) and (II), get:
$\angle BCP = \angle PBC$
BP = CP [If the angles subtended by two chords of a circle at the center are equal, the chords are equal]
As, P is on perpendicular of BC.
Hence, angle bisector of $\angle A$ and perpendicular bisector of BC intersect on the circumcircle of triangle ABC.

6. If two chords AB and CD of a circle AYDZBWCX intersect at right angle (see Fig.), prove that arc CXA + arc DZB = arc AYD + arc BWC = semicircle.

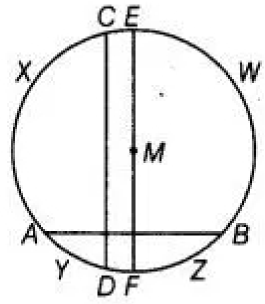


Solution:

Given: In the given circle AYDZBWCX, two chords AB and CD intersect at right angles.
To prove that arc CXA + arc DZB = arc AYD + arc BWC = Semi-circle.

Construction: Draw a diameter EF parallel to CD having centre M.
Proof: As, CD||EF
arc EC = arc PD … (I)
arc ECXA = arc EWB [symmetrical about diameter of a circle]
arc AF = arc BF …(II)

Also, know that: arc ECXAYDF = Semi-circle



Arc EA + arc AF = Semi-circle
Arc EC + arc CXA + arc FB = Semi-circle [From equation (II)]
Arc DF + arc CXA + arc FB = Semi-circle [From equation (I)]
Arc DF + arc FB + arc CXA = Semi-circle
Arc DZB + arc CXA = Semi-circle

As we know that, circle divides itself in two semi-circles, therefore the remaining portion of the circle is also equal to the semi-circle.
So, arc AYD + arc BWC = Semi-circle
Hence, proved.

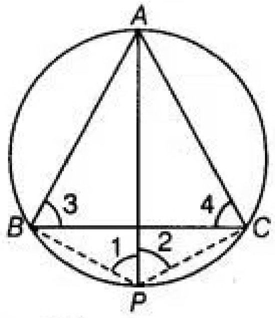
7. If ABC is an equilateral triangle inscribed in a circle and P be any point on the minor arc BC which does not coincide with B or C, prove that PA is angle bisector of ∠BPC.

Solution:

Given ΔABC is an equilateral triangle inscribed in a circle and P be any point on the minor arc BC which does not coincide with B or C.

To prove that PA is an angle bisector of ∠BPC.

Construction: Join PB and PC.



Proof: ABC is an equilateral triangle. So,

$\angle 3 = \angle 4 = 60°$

And, $\angle 1 = \angle 4 = 60°$ [Angle in the same segment AB]

Now, $\angle 2 = \angle 3 = 60°$ [Angle in the same segment AC]

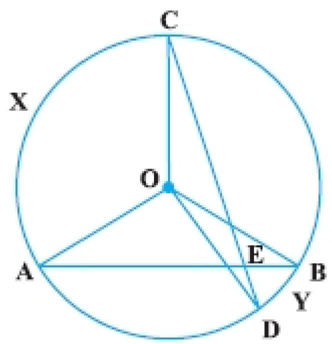
Also, $\angle 1 = \angle 2 = 60°$

Therefore, PA is the bisector of triangle BPC.

Hence, prove.

8. In Fig., AB and CD are two chords of a circle intersecting each other at point E. Prove that

$$\angle AEC = \frac{1}{2}\begin{pmatrix}\text{Angle subtended by arc CXA at centre} \\ +\text{angle subtended by arc DYB at the centre}\end{pmatrix}$$



Solution:

Given: AB and CD are two chords of a circle intersecting each other at point E.

To prove that $\angle AEC = \frac{1}{2}$ [Angles subtended by an arc CXA at the centre + angle subtended by arc DYB at the centre]

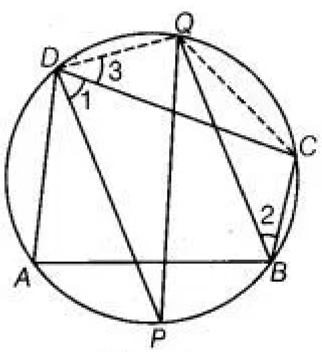
Construction: Join AC, BC and BD.
As we known that, the angles subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle, now arc CXA subtends $\angle AOC$ at the centre and $\angle ABC$ at the remaining part of the circle. So,
$\angle AOC = 2\angle ABC$ ...(I)
Also, $\angle BOD = \angle BCD$ ...(II)

Now, adding equation (I) and (II), get:
$\angle AOC + \angle BOD = 2(\angle ABC + \angle BCD)$ ...(III)

As, exterior angle of a triangle is equal to the sum of interior opposite angles. So, in triangle CEB:
$\angle AEC = \angle ABC + \angle BCD$ ...(IV)
Now, from equation (III) and (IV), get:
$\angle AOC + \angle BOD = 2\angle AEC$
$\angle AEC = \frac{1}{2}(\angle AOC + \angle BOD)$ or
$\angle AEC = \frac{1}{2}(\text{Angle subtended by an arc CXA at the centre +angle subtended by an arc DYB at the centre})$
Hence, proved.

9. If bisectors of opposite angles of a cyclic quadrilateral ABCD intersect the circle, circumscribing it at the points P and Q, prove that PQ is a diameter of the circle.

Solution:
Given: ABCD is a cyclic quadrilateral.
DP and QB are the bisectors of ∠D and ∠B, respectively.
To prove that PQ is the diameter of a circle.
Construction: Join QD and QC.



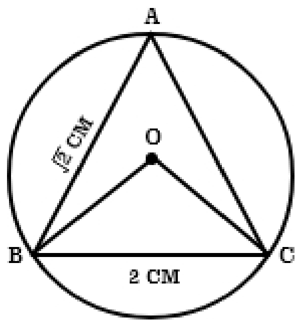
Proof: As, ABCD is a cyclic quadrilateral. So,
$\angle CDA + \angle CBA = 180°$ [Sum of opposite angles of cyclic quadrilateral is $180°$]
Now, dividing the above equation by 2, get:
$\frac{1}{2}\angle CDA + \frac{1}{2}\angle CBA = \frac{1}{2} \times 180°$

$\angle 1+\angle 2=90°$ ...(I) [As, $\angle 1=\frac{1}{2}\angle CDA$ and $\angle 2=\frac{1}{2}\angle CBA$]
$\angle 2=\angle 3$ [Angles in the same augment QC are equal] ...(II)
$\angle 1+\angle 3=90°$

Now, from equation (I) and (II), get:
$\angle PDQ=90°$
Hence, PQ is a diameter of a circle, because diameter of the circle.

10. A circle has radius $\sqrt{2}$ cm. It is divided into two segments by a chord of length 2 cm. Prove that the angle subtended by the chord at a point in major segment is 45°.

Solution:
Draw a circle having centre O and radius $\sqrt{2}$ cm. let chord BC, 2cm long divides the circle into two segments. And $\angle BAC$ lies in the major segment.



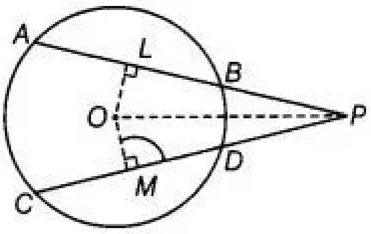
To prove that $\angle BAC=50°$
Construction: Join OB and OC.
$BC^2=(2)^2=4=2+2=(\sqrt{2})^2+(\sqrt{2})^2$
$BC^2=OB^2+OC^2$
In triangle BOC, get:
$BC^2=OB^2+OC^2$
So, $\angle BOC=90°$ [By converse of Pythagoras theorem]
Since, arc Bc subtends $\angle BOC$ at the centre O and $\angle BAC$ at the remaining part of the circle.
So,

$$\angle BAC=\frac{1}{2}\angle BOC$$
$$=\frac{1}{2}\times 90°$$
$$=45°$$

Hence, proved.

11. Two equal chords AB and CD of a circle when produced intersect at a point P. Prove that PB = PD.

Solution:
Given: Two equal chords AB and CD of a circle when produced intersect at a point P.
To prove that PB = PD.
Construction: Join OP and draw $OL \perp AB$ and $OM \perp CD$.



Proof: AB = CD
OL = OM [Equal chords are equidistance from the centre]

In triangle OLP and triangle OMP,
OL = OM [above prove]
$\angle OLP = \angle OMP$ [Each $90°$]
OP = OP [Common side]

So, $\Delta OLP \cong \Delta OMP$ [By RHS congruence rule]
LP = MP [By CPCT] …(I)
As, AB = CD

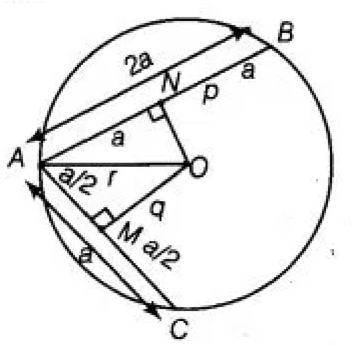
Now, dividing both side by 2 in the above equation, get:
$\frac{1}{2}AB = \frac{1}{2}CD$
BL = DM [Perpendicular distance from centre to the circle bisectors the chord that is AL = LB and CM = MD]

Now, subtracting equation (II) from equation (I), get:
LP – BL = MP – DM
Or PB =PD
Hence, proved.

12. AB and AC are two chords of a circle of radius *r* such that AB = 2AC. If *p* and *q* are the distances of AB and AC from the centre, prove that $4q^2 = p^2 + 3r^2$.

Solution:
Given: In a circle of radius r, there are two chords Ab and AC sch that AB = 2AC. Also, the distance of AB and AC from the centre are P and q, respectively.

To prove that $4q^2 = p^2 + 3r^2$



Proof: Suppose AC = a, then AB = 2a
At centre O, perpendicular is drawn to the chords AC and AB at M and N, respectively.

AM = MC = $\frac{a}{2}$

AN = NB = a

In triangle OAM,

$AO^2 = AM^2 + MO^2$ [By Pythagoras theorem]

$AO^2 = \left(\frac{a}{2}\right)^2 + q^2$ …(I)

In triangle OAN, using Pythagoras throrem:

$AO^2 = (AN)^2 + (NO)^2$

$AO^2 = (a)^2 + (p)^2$

From equation (I) and (II), get:

$\left(\frac{a}{2}\right)^2 + q^2 = a^2 + p^2$

$\frac{a^2}{4} + q^2 = a^2 + p^2$

$a^2 + 4q^2 = 4a^2 + 4p^2$

$4q^2 = 3a^2 + 4p^2$

$4q^2 = p^2 + 3(a^2 + p^2)$

$4q^2 = p^2 + 3r^2$ [In right angled triangle OAN, $r^2 = a^2 + p^2$]

Hence, proved.

13. In Fig., O is the centre of the circle, ∠BCO = 30°. Find *x* and *y*.